Noise Due to Pulse-to-Pulse Incoherence in Injection-Locked Pulsed Microwave Oscillators

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Abstract —It is demonstrated that partial pulse-to-pulse coherence in a pulsed oscillator system gives rise to an excess noise, which may be significantly reduced by injection locking.

I. Introduction

Injection locking of oscillators has become a standard technique in a variety of technically very important situations. In particular, it has been used to great advantage in connection with microwave negative resistance devices like, for example, the IMPATT diode [1]. In addition to its characteristic property of providing a stable oscillator output frequency, injection locking has also been used for many other purposes, like to generate, amplify, amplitude limit, and detect frequency-modulated or phase-modulated signals [1].

Another very important property of injection locking is its ability to suppress the inherent noise level of an oscillator. In a classical work in 1968, Kurokawa [2] analyzed how the oscillator noise could be improved by injection locking. This analysis has subsequently been improved and extended by several authors to include higher levels of injection signals and/or noise [3].

However, most treatments of noise in injection-locked oscillators consider the CW case and very little attention has been paid to the specific problems associated with pulsed operation [1]. Actually, the very pulsed nature of the operating mode is an additional source of noise, which may even dominate the inherent noise of the oscillator itself.

In a pulsed system, such as a pulsed radar transmitter system, the starting phases of the individual pulses are more or less randomly distributed. This partial pulse-to-pulse incoherence degrades the coherent superposition of pulses, which is crucial for obtaining high signal-to-noise ratios, and is manifested as an excess noise in the signal.

Thus in a pulsed system, injection locking plays a doubly beneficial role. Firstly, it will suppress the inherent noise level of the oscillator in the same way as for a CW system. Secondly, by locking the phases of the individual pulses, it will also decrease the excess noise due to partial pulse-to-pulse coherence.

This particular aspect of a pulsed system was touched upon in some early experiments on injection locking of pulsed oscillators [4], [5]. Furthermore, in a very recent paper [6], experimental results as well as a qualitative discussion were presented concerning noise in phase-primed solid-state pulsed radar transmitters. However, as far as we know, no further quantitative analysis has been given of the properties of this noise source and its suppression by injection locking.

The present work is meant to emphasize the importance of the problem and to present a more quantitative analysis of its main characteristics.

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II. Fourier Transform Results for a Pulsed System

We start our analysis by giving some elementary results for the Fourier transform of a pulsed system. If $f(t)$ denotes the (complex) amplitude variation of one pulse, we can write the amplitude variation $g(t)$ for a pulsed system consisting of $2N+1$ pulses as

$$g(t)=\sum_{k=-N}^{+N} f(t-kT)e^{i\phi_k} \tag{1}$$

where T is the pulse repetition time and we have included the possibility of different initial phases ϕ_k for consecutive pulses.

Taking the Fourier transform of (1), we obtain

$$G(\omega)=F(\omega)\sum_{k=-N}^{+N}\exp(-ik\omega T+i\phi_k) \tag{2}$$

where $F(\omega)$ denotes the transform of $f(t)$. The power spectrum $G_0(\omega)$, normalized with respect to the number of pulses, can be written

$$\frac{1}{2N+1}|G(\omega)|^2\equiv G_0(\omega)=|F(\omega)|^2H(\omega) \tag{3}$$

where $H(\omega)$ is given by

$$H(\omega)=\frac{1}{2N+1}\sum_{k,n=-N}^{+N}\exp[-i(k-n)\omega T+i(\phi_k-\phi_n)]. \tag{4}$$

We emphasize the fact that the normalized spectrum for the pulsed system is written as the product of the spectrum of the single pulse ($|F(\omega)|^2$) and a sampling function ($H(\omega)$).

Two extreme cases clearly illustrate the importance of the initial phases ϕ_k.

A. Complete Pulse-to-Pulse Coherence

If all pulses are initiated with the same phase, we have $\phi_k-\phi_n=0$ for all k and n and (4) reduces to

$$H(\omega)=H_0(\omega)\equiv\frac{1}{(2N+1)}\frac{\sin^2\left[\left(N+\frac{1}{2}\right)\omega T\right]}{\sin^2\frac{\omega T}{2}} \tag{5}$$

i.e., the well-known sampling function for a periodic, but finite, pulse train.

B. Complete Pulse-to-Pulse Incoherence

When all pulses are completely incoherent with initial phases randomly distributed, uniformly over the interval $[-\pi,\pi]$, we obtain after statistically averaging $H(\omega)$

$$\langle H(\omega)\rangle=1. \tag{6}$$

This implies that the spectrum of the pulsed system coincides with the spectrum of one pulse.

III. Partial Pulse-to-Pulse Coherence

Since a spectrum of a pulsed signal depends crucially on the coherence properties of consecutive pulses, we now consider the situation of partial pulse-to-pulse coherence. A technically important (and at the same time analytically simple) case is when all phases can be regarded as uncorrelated, but with a normally distributed deviation from the mean, i.e.,

$$\langle \phi_k \rangle = \phi_0$$
$$\langle \phi_k \phi_n \rangle = \langle (\Delta\phi)^2 \rangle \delta_{kn}. \tag{7}$$

In this case, the average of $H(\omega)$ can be written in the suggestive form

$$\left(\mu \equiv \exp\left(-\langle (\Delta\phi)^2 \rangle\right)\right)$$
$$\langle H(\omega) \rangle = \mu H_0(\omega) + (1-\mu) \tag{8}$$

i.e., as a weighted mean of a completely coherent part ($\mu H_0(\omega)$) and a completely incoherent part $(1-\mu)$, the weighting factor being determined by the rms phase error. We recognize that (8) describes the gradual transition between the previous limiting cases of completely coherent ($\langle (\Delta\phi)^2 \rangle \to 0, \mu \to 1$) and completely incoherent ($\langle (\Delta\phi)^2 \rangle \to \infty, \mu \to 0$) pulses.

The effect of decreasing degree of pulse-to-pulse coherence on $\langle H(\omega) \rangle$ is easily inferred from (8). As μ decreases, the maxima and minima of $\langle H(\omega) \rangle$ decrease and increase, respectively. We have

$$\max \langle H(\omega) \rangle \simeq 2\mu N$$
$$\min \langle H(\omega) \rangle = 1 - \mu. \tag{9}$$

In particular, the nonzero minimum of $\langle H(\omega) \rangle$ is equivalent to an excess white noise level between the high peaks of the coherent sampling function, which are now slightly reduced. The corresponding signal-to-noise ratio or peak-to-valley ratio s is simply [6]

$$S \simeq \frac{2\mu}{1-\mu} N = \frac{2N}{\exp\left(\langle (\Delta\phi)^2 \rangle\right) - 1}. \tag{10}$$

For small phase errors, we obtain

$$S \simeq \frac{2N}{\langle (\Delta\phi)^2 \rangle} \tag{11}$$

and taking, e.g., $2N = 60000$ and $\langle (\Delta\phi)^2 \rangle^{1/2} \simeq 1°$, we obtain a signal-to-noise ratio of $S \simeq 83$ dB.

IV. Effects of Injection Locking on Pulse-to-Pulse Coherence

Injection locking plays a very important role in suppressing the inherent oscillator noise in the outgoing signal [2]. This effect is well known for CW signals and will not be discussed further here. However, when injection locking strongly reduces the noise level in a pulsed system, see [7], this is achieved as a result of two independent effects.

1) The inherent oscillator noise is reduced as in the CW case.
2) The pulse-to-pulse coherence is improved.

For a solid-state microwave oscillator, the initial oscillator phases take on values which are randomly distributed over the interval $[-\pi, \pi]$. However, injection locking tends to lock the output signal to a certain phase ϕ_0, which is determined solely by the characteristic properties of the oscillator and the injected signal and which is independent of the initial phase of the oscillator.

The dynamic equation for the locking of the phase $\phi(t)$ of the output signal is [1, 2]

$$\frac{d\phi}{dt} = -\Delta\omega_0 - \Delta\omega_m \sin\phi \tag{12}$$

where $\Delta\omega_0$ is the difference between the frequencies of the locking signal and the free-running oscillator, and $\Delta\omega_m$ is the maximum frequency off-set for which locking can be achieved. $\Delta\omega_m$ is determined by the parameters of the oscillator together with the ratio of the amplitudes of the free-running oscillator and the injected signal.

The characteristic locking phase ϕ_0 is obtained from (12) as

$$\sin\phi_0 = -\frac{\Delta\omega_0}{\Delta\omega_m}. \tag{13}$$

For simplicity, we assume exact resonance ($\Delta\omega_0 = 0$, in which case the stable locking phase can be shown to be $\phi_0 = 0$ [1], [2]). The phase variation during the locking process is obtained by solving (12), assuming an initial phase ϕ_i. The solution becomes particularly simple for small ϕ_i, viz.

$$\phi \simeq \phi_i \exp(-\Delta\omega_m t). \tag{14}$$

Note that (14) implies a characteristic locking time $T_L = 1/\Delta\omega_m$.

We emphasize one consequence of (12)–(14), which is of particular importance in the present context. Phase locking is a dynamic process which continuously during pulses improves the pulse-to-pulse coherence by mapping the initial phase spread of 2π on a phase interval $2\Delta\phi(t)$ shrinking in time.

A more detailed analysis of noise due to partial pulse-to-pulse coherence, including the dynamics of the phase-locking process, will be presented in a later paper. At present, we will give a simplified model in terms of an effective phase spread. Since $\Delta\phi(t)$ varies between π and 0, we can, as a first estimate of the rms value of the phase deviation, take

$$\left[\langle (\Delta\phi)^2 \rangle\right]^{1/2} \simeq \pi \exp\left(-\tfrac{1}{2}\Delta\omega_m T_p\right) \tag{15}$$

where T_P is he length of the pulse. For simplicity, we have approximated the time variation of $\phi(t)$ as an exponential of the form given in (14) and taken the phase spread corresponding to half the pulse length. Using (15) in (11), we obtain the following expression for the noise level:

$$S \simeq \frac{2N}{\pi^2} \exp(+\Delta\omega_m T_p). \tag{16}$$

Taking, as before, $2N = 60000$ and a locking time $T_L = T_p/8$, we obtain $s \simeq 73$ dB.

Although the present analysis is rather qualitative on several important points, it does give an indication of the importance of pulse-to-pulse incoherence as a source of additional noise in pulsed oscillator systems.

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